CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, Jan./Feb. 2021
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. In which direction the directional derivative of x^2yz^3 is maximum at (2, 1, -1) and find the magnitude of this maximum. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). **(07 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. Evaluate $\int_C \vec{F}.\vec{dr}$ where $\vec{F} = xyi + (x^2 + y^2)j$ along the path of the straight line from (0, 0) to (1, 0) and then to (1, 1). **(06 Marks)**

b. Verify Green's theorem in a plane for $\int (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(07 Marks)**

c. Verify stoke's theorem for vector,
$\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(07 Marks)**

Module-2

3 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$. **(07 Marks)**

c. Solve : $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2 - 4x - 6$. **(07 Marks)**

OR

4 a. Solve : $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation of parameters. **(06 Marks)**

b. Solve : $x^2y'' + xy' + 9y = 3x^2 + \sin(3\log x)$. **(07 Marks)**

c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2} + w^2x = F\sin xt$, where w and F are constants. If at t = 0, x = 0 and $\frac{dx}{dt} = 0$, determine the motion when x = w. **(07 Marks)**

Module-3

5 a. Find the P.D.E. of the family of all spheres whose centres lie on the plane z = 0 and have a constant radius 'r'. **(06 Marks)**

b. Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$, when x = 0 and z = 0 if y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Find all possible solutions of one dimensional heat equations, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

OR

6 a. Solve : $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the conditions that z = 1 and $\frac{\partial z}{\partial x} = y$ when x = 0. **(06 Marks)**

b. Solve : $(y - z)p + (z - x)q = (x - y)$. **(07 Marks)**

c. Derive one dimensional wave equation in the standard form as, $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

Module-4

7 a. Discuss the nature of the series,
$\frac{2}{3} + \frac{2.3}{3.5} + \frac{2.3.4}{3.5.7} + \ldots\ldots$ **(06 Marks)**

b. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}.\cos x$ **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the values of a, b, c, d. **(07 Marks)**

OR

8 a. Discuss the nature of the series, $\sum_{n=1}^{\infty} \frac{(n+1)^n.x^n}{n^{n+1}}$ **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, prove that $\int_0^1 xJ_n(\alpha x).J_n(\beta x)dx = \frac{1}{2}[J_n'(\alpha)]^2$ if α = β. **(07 Marks)**

c. Using Redrigue's formula obtain expressions for $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$. **(07 Marks)**

Module-5

9 a. The Area of a circle (A) corresponding to diameter (D) is given below:

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the Area corresponding to diameter 105 using an appropriate interpolation formula. **(06 Marks)**

b. Find the cubic polynomial which passes through the points (2, 4), (4, 56), (9, 711), (10, 980) by using Newton's divided difference formula. **(07 Marks)**

c. Find the real root of the equation, $x \sin x + \cos x = 0$ near x = π using Newton's Raphson method. Carry out three iterations. **(07 Marks)**

OR

10 a. The following table gives the normal weights of babies during first eight months of life.

Age (in months)	0	2	5	8
Weight (in pounds)	6	10	12	16

Estimate the weight of the baby at the age of seven months using Lagrange's interpolation formula. **(06 Marks)**

b. Find the real root of $x\log_{10} x - 1.2 = 0$ by correct to four decimal places using Regula-Falsi method. **(07 Marks)**

c. Use Simpson's $\frac{3}{8}^{th}$ rule to obtain the approximate value of $\int_0^{0.3}(1-8x^3)^{\frac{1}{2}}dx$ by considering 3 equal intervals. **(07 Marks)**

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